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Propagation of chirped optical similaritons in inhomogeneous tapered centrosymmetric nonlinear waveguides doped with resonant impurities

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Abstract

We investigate the existence and propagation properties of linearly chirped self-similar solitons in an inhomogeneous tapered centrosymmetric nonlinear waveguide doped with resonant impurities under diffraction and nonlinearity management. The exact self-similar bright and dark soliton solutions are presented by employing an improved homogeneous balance principle and an F-expansion method. It is found that these chirped self-similar beams possess a strictly linear chirp that leads to efficient compression or amplification. The dynamical behaviors of these self-similar structures in a periodic distributed amplification waveguide system and an exponential diffraction decreasing waveguide are respectively studied for different choices of tapered index profile. The results show that the soliton shape and propagation behavior can be effectively controlled by selecting the diffraction, quintic nonlinearity, tapering, and gain or loss.

Keywords: chirp, similaritons, solitons

(Some figures may appear in colour only in the online journal)

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1. Introduction

Tapered graded-index waveguides are potentially useful in optical communications [1]. In these systems, tapering is a significant effect that plays a sensitive role on the light beams propagation [2–15]. For instance, it helps in maximizing light coupled into optical fibers, and integrated-optic devices and waveguides by reducing the reflection losses and mode mismatch effect [16, 17]. Apart from all its important applications, the tapered graded-index waveguide is an interesting object for studies, as it is a nonlinear system that can have a very rich dynamics. Remarkably, investigations of optical wave propagation inside in this important element have attracted more interest in recent years and several interesting results have been obtained [18–21].

The equation that governs the field dynamics in a tapered Kerr graded-index waveguide amplifier is a nonlinear Schrödinger-like equation that includes tareping and gain terms. Such model supports self-similar wave solutions of different types such as bright similaritons, Akhmediev breathers and rogue waves [16–23]. These self-similar structures are of primary importance because of their great value in understanding widely different physical phenomena [24]. Interestingly, self-similar waves (optical similaritons) may be useful in various real applications in fiber-optic telecommunications and photonics, since they can maintain their overall shapes but allow their amplitudes and widths to change according to the management of the system's parameters such as dispersion, nonlinearity, gain, inhomogeneity, and so on [25–27].

Recently, the research focus has been shifted towards nonlinear waveguides exhibiting higher order nonlinear effects. This should not be surprising because most of the practical cases have shown that the cubic model is not adequate for a realistic description of physical systems. In this setting, Ponomarenko and Haghgoo have developed a new model governing the propagation of an optical beam in a centrosymmetric nonlinear medium doped with resonant impurities in the limit of a large light carrier frequency detuning from the impurity resonance [28]. Noting that the resonant impurities could be rare-earth element atoms, erbium-doped glasses, or semiconductors doped with quantum dots (QDs). Considering the effect of tapering in the waveguide medium, this model has been recently extended to study the propagation behavior of chirped self-similar pulses by tailoring of the tapering function [29].

However, in realistic optical communications, the presence of nonuniformities that are due to various factors can profoundly alter the wave dynamics. Those nonuniformities often lead to such effects as the variable dispersion, phase modulation, and gain or loss [30]. Due to its potential applications, the nonlinear wave propagation in inhomogeneous systems has attracted special attention because these systems are considered to be more realistic than their corresponding homogeneous counterparts [31]. In the context of nonlinear optics, the first soliton dispersion management experiment in a fiber with hyperbolically decreasing group velocity dispersion was realized by Bogatyrev *et al* [32]. Moreover, the nonlinearity management in optics has been done experimentally using femtosecond pulses and layered Kerr media consisting of glass and air in [33].

In this paper, we consider the most general case, when the wave evolution occurs in a real tapered centrosymmetric nonlinear waveguide doped with resonant impurities whose diffraction, nonlinearity, and gain profile are allowed to change with the propagation distance. We will investigate the evolutions of chirped self-similar soliton solutions in the inhomogeneous nonlinear waveguide in the presence of all the distributed physical parameters. By using the improved homogeneous balance principle and the F-expansion method, we present exact self-similar solutions, including the bright and dark soliton solutions. The propagation dynamics of these self-similar solutions are discussed in the linearly chirped and unchirped cases by choosing different types of tapering profile. The results show that these self-similar optical structures can be generated and effectively controlled by modulating the system parameters.

The outline of this paper is as follows: in section 2 the generalized NLSE with varying diffraction, quintic nonlineariy, tapering, and gain or absorption is presented to model the optical beam propagation in an inhomogeneous tapered graded-index waveguide doped with resonant impurities. This section also describes the homogeneous balance principle and the F-expansion technique for the model under consideration, and introduces exact general self-similar wave solutions of the same. In section 3, we present the exact linearly chirped self-similar bright and dark soliton solutions and their characteristics. In section 4, we investigate the dynamics of self-similar solitons in a periodically distributed amplification waveguide and a diffraction decreasing waveguide. The final section is a summary of the main results.

2. Model equation and general self-similar solution

We are considering the following generalized quintic NLSE, representing optical beam propagation in an inhomogeneous tapered centrosymmetric nonlinear waveguide doped with resonant impurities:

$$i\frac{\partial\psi}{\partial z}+\frac{\beta(z)}{2}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{2}f(z)x^2\psi+\gamma(z)\left|\psi\right|^4\psi=i\frac{g(z)}{2}\psi \quad (1)$$

where z and x are the normalized dimensionless variables and ψ represents the complex envelope of the electric field. The functions $\beta(z)$ and $g(z)$ represent the diffraction coefficient and the net gain or loss, respectively. The inhomogeneous parameters $f(z)$ and $\gamma(z)$ denote tapering and quintic nonlinearity, respectively. Here all physical parameters β, f, γ and g are functions of the propagation distance z.

This equation describes the amplification (when $g(z)>0$) or attenuation (when $g(z)<0$) of light beams propagating nonlinearly inside an inhomogeneous tapered centrosymmetric nonlinear waveguides doped with resonant impurities. As previously noted, equation (1) with $\beta(z)=\gamma(z)=1$ and $g(z)=f(z)=0$ has been derived for the first time by Ponomarenko and Haghgoo to describe beam propagation in a centrosymmetric medium doped with resonant impurities whose resonant frequencies lie sufficiently far away from the beam carrier frequency [28]. When $\beta(z) = \gamma(z) = 1$ and $g(z) = 0$, equation (1) turns into the NLSE in [29]. To the best of our knowledge, the exact chirped self-similar soliton solutions of the inhomogeneous equation (1) with distributed diffraction, nonlinearity, tapering, and gain (loss) term have not been considered so far. Our goal here is to study the evolutions of chirped sel-similar solitons in the tapered graded-index nonlinear waveguide amplifier within the framework of the model (1).

Generally, equation (1) is not integrable. To solve this model, we begin our analysis by writing the complex function $\psi(z,x)$ in a polar form as

$$\psi(z,x) = A(z,x)\exp\left[i\Phi(z,x)\right], \tag{2}$$

where the amplitude $A(z,x)$ and phase $\Phi(z,x)$ are real functions of z and x. Substituting equations (2) into (1) and separating imaginary and real parts, we have

$$A_z + \frac{\beta}{2}\left(2A_x\Phi_x + A\Phi_{xx}\right) - \frac{g}{2}A = 0, \tag{3}$$

$$-A\Phi_z + \frac{\beta}{2}\left(A_{xx} - A\Phi_x^2\right) + \frac{1}{2}f\,x^2A + \gamma A^5 = 0. \tag{4}$$

According to the homogeneous balance principle and the F-expansion method [34, 35], the solutions of equations (3) and (4) can be selected as the following forms:

$$A(z,x) = h(z)F(\xi), \tag{5}$$

$$\Phi(z,x) = C(z)x^2 + \Gamma(z)x + \Omega(z), \tag{6}$$

$$\xi(z,x) = p(z)x + q(z) \tag{7}$$

where $C(z)$, $\Gamma(z)$, and $\Omega(z)$ are the parameters related to the phase-front curvature, the frequency shift, and the phase offset, respectively, to be determined. Parameter functions $p(z)$ and $q(z)$ are related to the width and the group velocity, respectively. We note that here the varying parameter $h(z)$ can be used to modulate the amplitude of self-similar waves. For the present problem, we assume that $F(\xi)$ in (5) satisfies the generalized auxiliary elliptic equation [36]:

$$F'^2 = c_0 + c_2F^2 + c_4F^4 + c_6F^6, \tag{8}$$

where $F' = dF/d\xi$, and c_0, c_2, c_4, and c_6 are all real constants.

Substituting expressions (5)–(8) into equations (3) and (4), collecting coefficients of power of x^kF^l and $\sqrt{c_0 + c_2F^2 + c_4F^4 + c_6F^6}$ $(k = 0, 1, 2; l = 0, 1, 2, 3)$ then setting each coefficients to zero, we obtain the following over-determined system of equations:

$$h_z + C\beta h - \frac{1}{2}gh = 0, \tag{9}$$

$$hp_z + 2C\beta ph = 0, \tag{10}$$

$$hq_z + \beta p\Gamma h = 0, \tag{11}$$

$$-h\Omega_z + \frac{1}{2}\beta p^2hc_2 - \frac{1}{2}\beta\Gamma^2h = 0, \tag{12}$$

$$\frac{3}{2}\beta hp^2c_6 + \gamma h^5 = 0, \tag{13}$$

$$\beta hp^2c_4 = 0, \tag{14}$$

$$h\Gamma_z + 2C\beta\Gamma h = 0, \tag{15}$$

$$hC_z + 2\beta hC^2 - \frac{1}{2}f\,h = 0 \tag{16}$$

where the subscript means the derivative with respect to z. Solving equations (9)–(16) self-consistently, one obtains the parameter functions:

$$C(z) = \frac{W_z}{2\beta W}, \tag{17}$$

$$\Gamma(z) = \frac{\Gamma_0}{W}, \tag{18}$$

$$\Omega(z) = \Omega_0 - \frac{\left(\Gamma_0^2 - c_2p_0^2\right)}{2}\int_0^z \frac{\beta(s)}{W^2(s)}ds, \tag{19}$$

$$p(z) = \frac{p_0}{W}, \tag{20}$$

$$q(z) = q_0 - p_0\Gamma_0\int_0^z \frac{\beta(s)}{W^2(s)}ds, \tag{21}$$

$$h(z) = \frac{h_0}{\sqrt{W}}\exp\left[\frac{1}{2}\int_0^z g(s)ds\right] \tag{22}$$

and the conditions on tapering, gain and width functions:

$$\beta W_{zz} - \beta_z W_z = f(z)\beta^2 W, \tag{23}$$

$$g(z) = \frac{1}{2}\left(\frac{\gamma\beta_z - \beta\gamma_z}{\gamma\beta}\right) \tag{24}$$

where the subscript 0 denotes the initial values of the corresponding parameters at distance $z = 0$, and $W(z)$ is the width of the self-similar wave. We also obtain the condition $c_4 = 0$ from equation (14).

Incorporating these results back into equation (2), we derive the general self-similar wave solutions to the inhomogeneous NLSE given in equation (1) as

$$\psi(z,x) = \frac{h_0}{\sqrt{W}}\exp\left[\frac{1}{2}\int_0^z g(s)ds\right]F(\xi)\exp\left[i\Phi(z,x)\right], \tag{25}$$

where the similarity variable and phase are given respectively as

$$\xi = \frac{p_0}{W(z)}x - p_0\Gamma_0\int_0^z \frac{\beta(s)}{W^2(s)}ds + q_0, \tag{26}$$

$$\Phi(z,x) = \frac{W_z}{2\beta W}x^2 + \frac{\Gamma_0}{W}x - \frac{\left(\Gamma_0^2 - c_2p_0^2\right)}{2}\int_0^z \frac{\beta(s)}{W^2(s)}ds + \Omega_0 \tag{27}$$

with $F(\xi)$ satisfying equation (8).

Equation (25) together with the relations (26) and (27) are the central theoretical results representing the general form of exact linearly chirped self-similar solutions for the GNLSE (1) subject to constraints (23) and (24) on the physical parameters. These important results show the universal influence of the width parameter $W(z)$ on the properties of wave solutions. It influences not only the form of the amplitude, the phase, and the group velocity, but also the curvature of the wavefront, the spatial frequency shift, and the homogeneous phase shift. We mention that for different tapering profiles $F(z)$, $W(z)$ can be obtained from equation (23) for a given diffraction management function $\beta(z)$. Remarkably, the phase function (27) indicates that the self-similar beams obtained here are linearly chirped. This chirping property is very interesting from a practical point of view.

It is noteworthy that the general self-similar solution (25) can be rewritten as (if setting $h_0 = p_0 = 1$):

$$\psi(z,x) = \frac{1}{\sqrt{W(z)}} F\left[\frac{x - x_g(z)}{W(z)}\right] e^{i\Phi(z,x)+G(z)}, \tag{28}$$

where $G(z) = \frac{1}{2}\int_0^z g(s)ds$ and $x_g(z)$ signify the guiding-center position given by

$$x_g(z) = W(z)\left(\Gamma_0 \int_0^z \frac{\beta(s)}{W^2(s)} ds - q_0\right). \tag{29}$$

Obviously, this form of the self-similar wave solution is more general than the known result [28, 29] because of the appearance of distributed diffraction, nonlinearity, and the loss gain coefficients parameters in the the beam's parameters; and thus it also provides a possible way to tune experimentally the self-similar wave by selecting different forms of them. If one can determine the functions $F(\xi)$ from the nonlinear ordinary differential equation (8), then we can construct the self-similar solutions of the class of NLSE under investigation based on the general wave form (25). It is well known that the auxiliary ordinary differential equation (8) is exactly solved and has localized solutions, given by bright and dark solitons (at $c_0 = 0$) [37, 38]. As shown previously, the condition $c_4 = 0$ is required here, which leads equation (8) to be more difficult for solving than that of the corresponding equation with $c_4 \neq 0$. In the following, we find two types of closed form solutions of this equation that we will use to construct the exact self-similar wave solutions of equation (1) exhibiting interesting nontrivial behavior.

3. Chirped self-similar soliton solutions

Below, the explicit localized chirped self-similar solutions of equation (1) by employing the closed form solutions of equation (8) (for the case $c_4 = 0$) and the general waveform (25) are presented in two types.

3.1. Chirped self-similar bright solitons

We have find a self-similar bright soliton solution for equation (1) in the following form:

$$\psi(z,x) = \frac{h_0}{\sqrt{W}} \exp\left[\frac{1}{2}\int_0^z g(s)ds\right] A\,\mathrm{sech}^{1/2}(\mu\xi)\exp[i\Phi(z,x)], \tag{30}$$

where

$$A = \left(-\frac{c_2}{c_6}\right)^{1/4}, \mu = 2\sqrt{c_2} \tag{31}$$

with $c_2 > 0$ and $c_6 < 0$. Here the constant c_0 takes the value $c_0 = 0$.

3.2. Chirped self-similar dark solitons

We have find another solution, representing a chirped self-similar dark soliton given by the following expression:

$$\psi(z,x) = \frac{h_0}{\sqrt{W}} \exp\left[\frac{1}{2}\int_0^z g(s)ds\right]\left(\frac{B\tanh(\eta\xi)}{\sqrt{3-\tanh^2(\eta\xi)}}\right)\exp[i\Phi(z,x)], \tag{32}$$

where

$$B = \sqrt{2}\left(-\frac{c_2}{3c_6}\right)^{1/4}, \eta = \sqrt{-c_2} \tag{33}$$

with $c_2 < 0$ and $c_6 > 0$. It should also be emphasized that the constants c_0, c_2 and c_6 have the following relation: $c_0 = -\frac{2c_2}{3}\sqrt{-\frac{c_2}{3c_6}}$.

Having obtained the exact self-similar soliton solutions of equation (1), our next aim is to analyze their propagation characteristics under diffraction and nonlinear management by considering various forms for both varied diffraction and nonlinearity parameters which turn out to be of physical relevance.

4. Dynamical behavior of chirped self-similar solitons

Now we shall discuss the dynamical properties of the obtained self-similar solitons for the various choices of the system parameters. We observed that self-similar beams propagation and formation in the present waveguide can occur when five functions $\beta(z)$, $f(z)$, $\gamma(z)$, $g(z)$, and $W(z)$ satisfy the conditions (23) and (24). In the following, the soliton propagation is studied in a tapered ($f(z) \neq 0$) and an untapered waveguide ($f(z) = 0$) under varying diffraction and nonlinearity profiles.

4.1. CASE-I: periodically distributed amplification waveguide

As a first example, we discuss the evolutional behaviour of the self-similar beams in a periodically distributed waveguide whose diffraction parameter is distributed according to [39]

$$\beta(z) = \beta_0\cos(\sigma_0 z), \tag{34}$$

and take the quintic nonlinearity parameter $\gamma(z)$ as

$$\gamma(z) = \gamma_0\cos(\sigma_0 z)e^{-\kappa z}, \tag{35}$$

where β_0, σ_0 and γ_0 are arbitrary constants. In this situation, the gain (loss) distributed function (24) is of the constant form $g(z) = \kappa/2$, which corresponds to the diffraction decreasing (increasing) waveguide for $\kappa < 0$ ($\kappa > 0$).

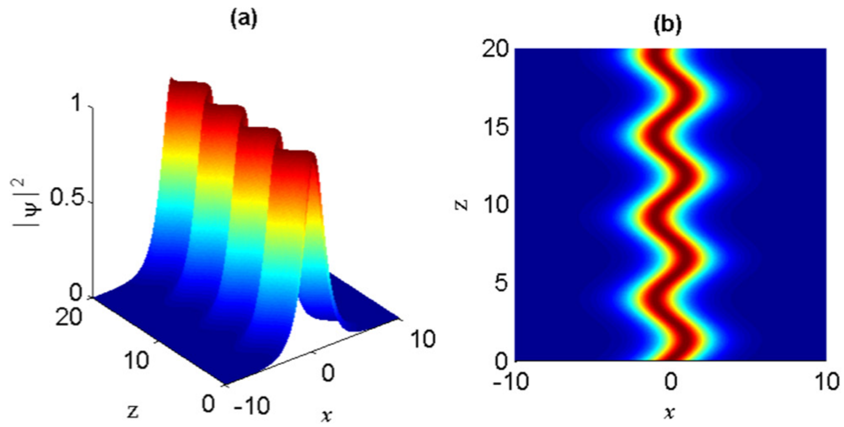

Figure 1. Intensity distribution of (a) and (b) the bright soliton given by equation (30) with $c_2 = \frac{1}{4}$, $c_6 = -\frac{1}{4}$, $c_0 = 0$. Other parameters are $h_0 = 1$, $\Gamma_0 = 2$, $\beta_0 = 0.6$, $\sigma_0 = 1.2$, $q_0 = 0$, $\kappa = 0$, and $C_0 = 0$, and $p_0 = 1$.

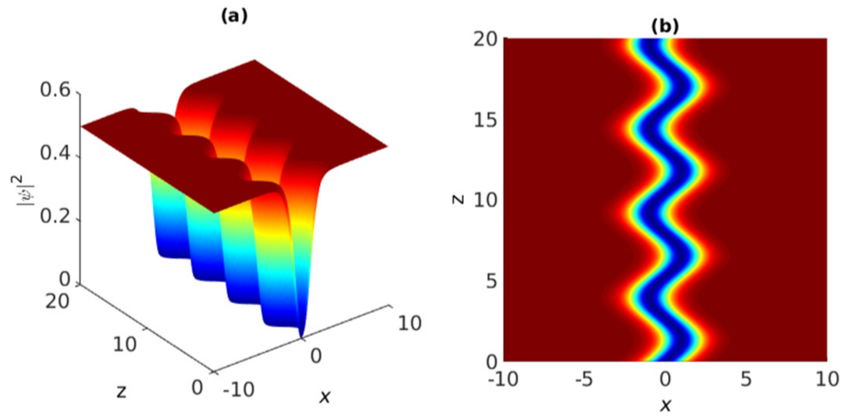

Figure 2. Intensity distribution of (a) and (b) the dark soliton given by equation (32) with $c_2 = -1$, $c_6 = \frac{4}{3}$, $c_0 = \frac{1}{3}$. Other parameters are the same as that of figure 1.

We concentrate here on the specific case of self-similar solitons propagation in an untapered periodically distributed waveguide, that is $f(z) = 0$. In this case, the beam width can be obtained from equation (23) as:

$$W(z) = W_0 \left(1 + C_0 \int_0^z \beta(s)ds\right), \tag{36}$$

where W_0 and C_0 are constants related to the initial beam width and the initial chirp, respectively. In the following, without the loss of generality, we will let $W_0 = W(0) = 1$ [16, 40]. We notice that the width of the beam vary linearly with the propagation distance z in the case of a constant diffraction system. Especially if $\beta(z) = 1$ and the phase chirp C in (6) is replaced by $-C$, the similariton width (36) can be readily reduced to the one in [28].

Inserting equations (36) into (27), one obtains the phase function as

$$\Phi(z,x) = \frac{C_0}{1 + C_0 \int_0^z \beta(s)ds}\frac{x^2}{2} + \frac{\Gamma_0}{1 + C_0 \int_0^z \beta(s)ds}x - \frac{\left(\Gamma_0^2 - c_2 p_0^2\right)}{2}\int_0^z \frac{\beta(s)}{\left(1 + C_0 \int_0^z \beta(s)ds\right)^2}ds + \Omega_0. \tag{37}$$

From these results, we can see that the accumulated diffraction $D(z) = \int_0^z \beta(s)ds$ influences the form of the width and phase and consequently the amplitude, the chirp, and the group velocity parameters.

The evolution of the intensity distribution for the self-similar bright soliton solution (30) and dark solution (32) are illustrated for the case of unchirped beams ($C_0 = 0$) in figures 1 and 2, respectively. The results for the chirped self-similar solitons ($C_0 \neq 0$) are shown respectively in figures 3 and 4. On comparing these figures, we can clearly see that the chirp C_0 leads to the periodical change in the intensity of the self-similar structures. One can also see the snake-like behavior of the intensity distributions in the case of unchirped beams (figures 1 and 2). For this case, the soliton's center is oscillating with the propagation distance. It should be noted that the transverse oscillations of the self-similar waves intensity result especially from $p_0\Gamma_0 \int_0^z \frac{\beta(s)}{W^2(s)}ds$ in equation (26).

4.2. CASE-II: diffraction decreasing optical waveguide

Let us consider another distributed system, which is a realistic optical waveguide with decreasing diffraction as follows [39]

$$\beta(z) = -\sigma_0 \exp(\sigma_0 z), \tag{38}$$

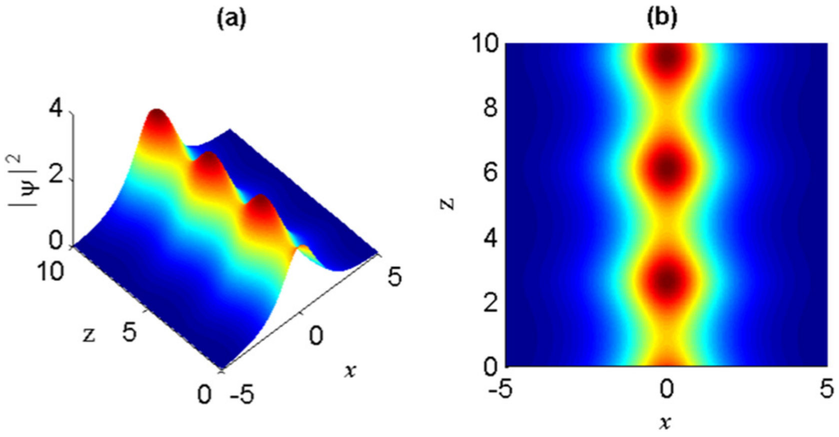

Figure 3. Intensity distribution of (a) and (b) the bright soliton given by equation (30) with $c_2 = 0.01$, $c_6 = -1$, $c_0 = 0$. Other parameters are $h_0 = 0.2$, $\Gamma_0 = 0$, $\beta_0 = 1.8$, $\sigma_0 = 1.8$, $q_0 = 0$, $\kappa = 0$, and $p_0 = 0.01$, and $C_0 = 0.2$.

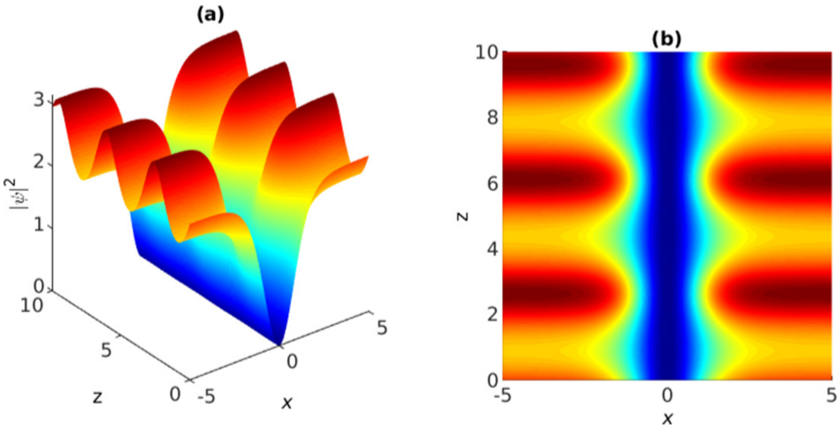

Figure 4. Intensity distribution of (a) and (b) the dark soliton given by equation (32) with $c_2 = -4$, $c_6 = \frac{16}{3}$, $c_0 = \frac{4}{3}$. Other parameters are the same as that of figure 3.

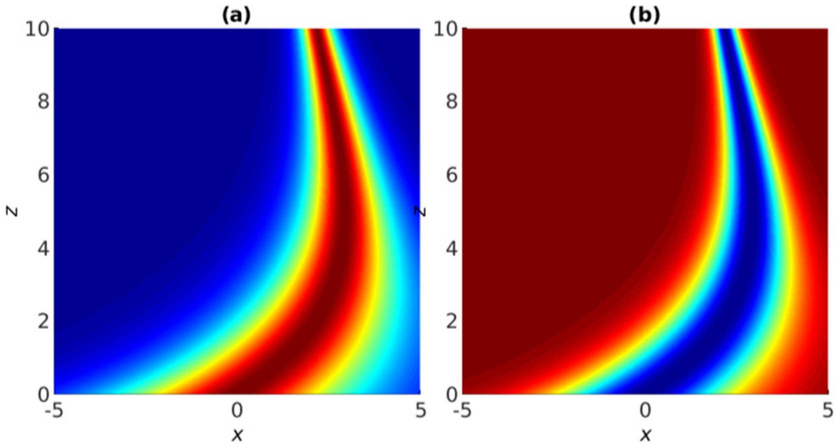

Figure 5. Contour plot depicting the nonlinear compression of (a) the bright soliton given by equation (30) with $c_2 = 0.025$, $c_6 = -0.1$, $c_0 = 0$ and (b) the dark soliton given by equation (32) with $c_2 = -1$, $c_6 = 2.65$, $c_0 = 0.236$. Other parameters are $h_0 = 0.141$, $\Gamma_0 = 1$, $q_0 = 0$, $p_0 = 1$, $\sigma_0 = -0.4$, and $\sigma_1 = -0.4$.

and quintic nonlinearity parameter $\gamma(z)$ as

$$\gamma(z) = -\sigma_1 \exp(\sigma_1 z) \tag{39}$$

where σ_0 and σ_1 are real constants (with $\sigma_0 < 0$ for diffraction deceasing waveguides).

We concentrate especially on analyzing the propagation characteristics of the self-similar solitons when the nonlinear waveguide exhibits the tapering effect. Noting that the existence of the tapering function $f(z)$ makes the solution of equation (23) more difficult than that of the corresponding equation in the case of $f(z)=0$. Here, the shape of the taper $f(z)$ is chosen to take an exponential profile of the form $f(z)=f_0\exp(-\sigma_0 z)$, which in the limit $z\to 0$, tends toward $f(z)=f_0$ that corresponds to the case of constant tapered waveguide [29]. Then, we can find the beam's width $W(z)=W_0\exp\left(\frac{\sigma_0}{2}z\right)$ as can be obtained from the condition (23). This type of width was used by Dai *et al* for studying the dynamical behaviors of similaritons in a realistic cubic-quintic waveguide with hyperbolically decreasing diffraction [41]. To satisfy the parametric condition (23), we set $f_0=\sigma_0/4$. Accordingly, the distributed gain/loss function $g(z)$ can be obtained exactly through equation (24) as $g(z)=(\sigma_0-\sigma_1)/2$ (with $\sigma_0>\sigma_1$ for the gain and $\sigma_0<\sigma_1$ for the loss).

The propagation dynamics of the self-similar bright soliton (30) and dark soliton (32) solutions with diffraction and nonlinearity coefficients given above are presented in figures 5(a) and (b) respectively. As it is seen from this figure, the width of the two solitons gets compressed along propagation distance, which can be proved useful in practical applications.

Stability of these chirped self-similar solitons with respect to some perturbations is a significant issue, because only stable solitons are promising for experimental observations and physical applications. It may be amplitude perturbation, random noises and the slight violation of the parametric conditions. We point out that stability analyses can be done by numerical simulations and the linear stability theory of the solutions with perturbations initially implanted. In the present study, we have found that the obtained self-similar localized structures characteristically exist due to a balance among diffraction, quintic nonlinearity, tapering, and gain or loss. The stability aspects of such privileged closed form solutions typically require detailed individual analysis based on such balance aspects. Detailed stability analyses are now under investigation.

5. Conclusions

In this paper, we have investigated an inhomogeneous nonlinear Schrödinger equation with diffraction and quintic nonlinearity in the presence of tapering and gain or loss, which governs the optical beam propagation in an inhomogeneous tapered centrosymmetric nonlinear waveguide doped with resonant impurities. We have obtained exact chirped self-similar bright and dark soliton solutions of the model, using the homogeneous balance principle and the F-expansion technique. Conditions on the inhomogeneous waveguide parameters for the existence of these self-similar structures are also found. We have discussed the propagation behaviors of self-similar solitons in a periodic distributed waveguide system and an exponential diffraction decreasing waveguide. It is observed that the self-similar wave structure and dynamical behavior can be controlled by choosing the system parameters appropriately. The obtained chirped structures can be used in various applications in pulse compression or amplification and thus they are particularly useful in the design of nonlinear waveguide amplifiers, optical pulse compressors, and solitary-waves propagating in tapered centrosymmetric nonlinear waveguides doped with resonant impurities. In addition, these results may contribute to improve the understanding of physical processes in tapered centrosymmetric waveguides such as the interactions between different types of nonlinear localized waves.

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The authors also declare that there is no conflict of interest.

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