

Conservation laws for cubic–quartic optical solitons with complex Ginzburg–Landau equation having five nonlinear refractive index structures

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This paper recovers and exhibits the conservation law for cubic–quartic complex Ginzburg–Landau model which are considered with five nonlinear forms. The Hamiltonian, linear momentum and the power are derived, save one. For parabolic–nonlocal law, the Hamiltonian and the linear momentum are not retrievable by Lie symmetry.

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1. Introduction

The dynamics of optical soliton propagation is described by various forms of nonlinear evolution equations that are in the complex domain. One of the most viable models is the complex Ginzburg–Landau equation (CGLE) that has been extensively addressed all across the board [1–15]. The latest form of CGLE is pure–cubic (PC) CGLE which is considered when chromatic dispersion (CD) is low and hence third–order dispersion (3OD) replaces it. This model has been employed and its conservation laws have been recovered [3, 14]. Later, this PC–CGLE is extended to cubic–quartic (CQ) CGLE where in addition to 3OD, fourth–order dispersion (4OD) comes in and, thus the combination of 3OD and 4OD effects compensates for the low count of CD. The current paper is consequently a follow up of the previously reported results during 2021 where soliton solutions to CQ–CGLE were reported [2]. The current work now reports conservation laws that emerge from CQ–CGLE. The conserved quantities are yielded from Lie symmetry methodology and the conserved quantities are finally computed and exhibited for the five nonlinear forms that are studied.

1.1. Governing model

The CQ–CGLE with Hamiltonian perturbation terms is structured as:

$$\begin{aligned}
 iq_t + iaq_{xxx} + bq_{xxxx} + F(|q|^2)q &= \alpha \frac{|q_x|^2}{q^*} \\
 + \frac{\beta}{4|q|^2 q^*} [2|q|^2(|q|^2)_{xx} - \{(|q|^2)_x\}^2] + \gamma q \\
 + i \left[\lambda (|q|^{2m} q)_x + \delta q_x + \nu |q|^{2m} q_x \right] &+ \mu (|q|^{2m})_{xx} q. \quad (1)
 \end{aligned}$$

Here F stands for the real-valued nonlinear forms. λ and δ imply to the coefficient of self-steepening and inter–modal dispersion sequentially. γ purports the coefficient of detuning effect and $i = \sqrt{-1}$. α and β are the coefficients of perturbation effects. μ and ν correspond to the coefficients of nonlinear dispersion. a and b signify the coefficient of 3OD and 4OD respectively and the first term in (1) implies to linear temporal evolution. $q = q(x, t)$ purports the complex-valued wave profile. t purports the time in

dimensionless form and x accounts for the non-dimensional distance. Finally m stands for maximum intensity parameter.

2. Conservation laws

For the conserved flow, we assume $q = u + iv$ to recover a real system that is a closed form of the respective partial differential equations whose conserved flows (T^t, T^x) are derived by the multiplier methodology. The multiplier $Q = (-u, v)$ gives the ‘power’ conserved density

$$T_1^t = \frac{v^2 + u^2}{2}. \quad (2)$$

In each case, the ‘power’ conserved density is formulated to be:

$$\Phi_1^t = |q|^2. \quad (3)$$

Also, if

$$\lambda = -\mu, \quad (4)$$

together with

$$\beta = \alpha = 0, \quad (5)$$

we get Hamiltonian and linear momentum for (2)-(5). In each case, the **linear momentum** density is

$$T_2^t = \frac{v_x u + u_x v}{2}, \quad (6)$$

and the **momentum density** for the systems above is

$$\Phi_2^t = \mathcal{J}(q^* q_x). \quad (7)$$

Therefore the Hamiltonian (H), linear momentum (M) and the power (P) for the various nonlinear forms are exhibited in the current paper.

2.1. Kerr law

The CQ–CGLE with Kerr nonlinear form is written as:

$$\begin{aligned} i q_t + i a q_{xxx} + b q_{xxxx} + c |q|^2 q &= \alpha \frac{|q_x|^2}{q^*} \\ &+ \frac{\beta}{4 |q|^2 q^*} [2 |q|^2 (|q|^2)_{xx} - \{(|q|^2)_x \}^2] + \gamma q \\ &+ i \left[\frac{\lambda (|q|^2 q)_x + \delta q_x + \nu |q|^2 q_x}{+\mu (|q|^2)_x q} \right]. \end{aligned} \quad (8)$$

The bright soliton to the CQ–CGLE (8) is structured as [2]:

$$q(x, t) = A \operatorname{sech}[B(x - vt)] e^{i(-\kappa x + \omega t + \theta_0)}. \quad (9)$$

Here A purports the amplitude of the soliton, while B signifies its inverse width. The parameters κ and ω represent frequency and wave number sequentially while θ_0 stands for the phase constant. The density of Hamiltonian is

$$\begin{aligned} T_3^t &= \frac{\mu u_x v^3}{4} - \frac{\nu v u_x u^2}{4} + \frac{v \mu u_x u^2}{4} - \frac{u \mu v_x v^2}{4} \\ &+ \frac{\nu v_x u^3}{4} - \frac{\nu u_x v^3}{4} - \frac{\mu v_x u^3}{4} + \frac{c v^4}{4} + \frac{c u^4}{4} \\ &+ \frac{u v v_x v^2}{4} + \frac{c v^2 u^2}{2} + \frac{v a u_{xxx}}{2} + \frac{v b v_{xxx}}{2} + \frac{u b u_{xxx}}{2} \\ &- \frac{\gamma v^2}{2} - \frac{v \delta u_x}{2} + \frac{u \delta v_x}{2} - \frac{u a v_{xxx}}{2} - \frac{\gamma u^2}{2}, \end{aligned} \quad (10)$$

so that

$$\begin{aligned} \Phi_3^t &= \frac{b \mathcal{R}(q q_{xxxx})}{2} - \frac{a \mathcal{J}(q^* q_{xxx})}{2} + \frac{c |q|^4}{4} - \frac{\gamma |q|^2}{2} \\ &- \frac{\mu \mathcal{J}(q^* q_x) |q|^2}{4} + \frac{\delta \mathcal{J}(q^* q_x)}{2} + \frac{\nu \mathcal{J}(q^* q_x) |q|^2}{4}. \end{aligned} \quad (11)$$

The conserved quantities are thus given as:

$$P = \int_{-\infty}^{\infty} |q|^2 dx = \frac{2A^2}{B}, \quad (12)$$

$$M = i a \int_{-\infty}^{\infty} (q^* q_x - q q_x^*) dx = \frac{2a \kappa A^2}{B}, \quad (13)$$

$$\begin{aligned} H &= \int_{-\infty}^{\infty} \Phi_3^t dx \\ &= \int_{-\infty}^{\infty} \left[\frac{a(q_x^* q_{xx} - q_x q_{xx}^*)}{4i} + \frac{b |q_{xxx}|^2}{2} + \frac{c |q|^4}{4} - \frac{\gamma |q|^2}{2} \right. \\ &\left. + \frac{\delta(q^* q_x - q q_x^*)}{4i} - \frac{(\mu - \nu)(q^* q_x - q q_x^*) |q|^2}{8i} \right] dx \\ &= \frac{\kappa(\mu - \nu)A^4}{3B} - \frac{A^2(\delta \kappa + \gamma)}{B} - \frac{2a \kappa(B^2 + \kappa^2)A^2}{B} \\ &\quad + \frac{b(7B^4 + 2\kappa^2 B^2 + \kappa^4)A^2}{B} + \frac{cA^4}{3B}. \end{aligned} \quad (14)$$

2.2. Parabolic law

The CQ–CGLE with parabolic nonlinear form is given by:

$$\begin{aligned} i q_t + i a q_{xxx} + b q_{xxxx} + (c_1 |q|^2 + c_2 |q|^4) q &= \alpha \frac{|q_x|^2}{q^*} \\ &+ \frac{\beta}{4 |q|^2 q^*} [2 |q|^2 (|q|^2)_{xx} - \{(|q|^2)_x \}^2] + \gamma q \end{aligned}$$

$$+i \left[\begin{array}{c} \lambda(|q|^2 q)_x + \delta q_x + \nu |q|^2 q_x \\ + \mu(|q|^2)_x q \end{array} \right]. \quad (15)$$

The structure of the bright soliton to the CQ–CGLE (15) is the same as that of Kerr nonlinear form given by (9).

$$\begin{aligned} T_3^t &= \frac{v^6 c_2}{6} + \frac{v^2 u^4 c_2}{2} + \frac{u^6 c_2}{6} + \frac{u^2 c_2 v^4}{2} - \frac{u \mu v_x v^2}{4} \\ &+ \frac{u v v_x v^2}{4} + \frac{\mu u_x v^3}{4} + \frac{v^2 c_1 u^2}{2} + \frac{\nu v_x u^3}{4} + \frac{v \mu u_x u^2}{4} \\ &- \frac{\nu v u_x u^2}{4} - \frac{\nu u_x v^3}{4} + \frac{u^4 c_1}{4} + \frac{v^4 c_1}{4} - \frac{\mu v_x u^3}{4} \\ &+ \frac{u b u_{xxxx}}{2} + \frac{u \delta v_x}{2} + \frac{v a u_{xxx}}{2} - \frac{\gamma v^2}{2} \\ &+ \frac{v b v_{xxxx}}{2} - \frac{v \delta u_x}{2} - \frac{u a v_{xxx}}{2} - \frac{\gamma u^2}{2}, \quad (16) \end{aligned}$$

so that

$$\begin{aligned} \Phi_3^t &= \frac{b \mathcal{R}(q q_{xxxx}^*)}{2} - \frac{a \mathcal{J}(q^* q_{xxx})}{2} \\ &+ \frac{c_1 |q|^4}{4} + \frac{c_2 |q|^6}{6} - \frac{\gamma |q|^2}{2} - \frac{\mu \mathcal{J}(q^* q_x) |q|^2}{4} \\ &+ \frac{\delta \mathcal{J}(q^* q_x)}{2} + \frac{\nu \mathcal{J}(q^* q_x) |q|^2}{4}. \quad (17) \end{aligned}$$

With the same structure of the soliton, linear momentum and the power stays the same as that of Kerr nonlinear form while the Hamiltonian is given by:

$$\begin{aligned} H &= \int_{-\infty}^{\infty} \Phi_3^t dx \\ &= \int_{-\infty}^{\infty} \left[\frac{a(q_x^* q_{xx} - q_x q_{xx}^*)}{4i} + \frac{b |q_{xx}|^2}{2} + \frac{c_1 |q|^4}{4} \right. \\ &+ \frac{c_2 |q|^6}{6} - \frac{\gamma |q|^2}{2} + \frac{\delta(q^* q_x - q q_x^*)}{4i} \\ &\left. - \frac{(\mu - \nu)(q^* q_x - q q_x^*) |q|^2}{8i} \right] dx \\ &= \frac{\kappa(\mu - \nu) A^4}{3B} - \frac{A^2(\delta \kappa + \gamma)}{B} - \frac{2a\kappa(B^2 + \kappa^2) A^2}{B} \\ &+ \frac{b(7B^4 + 2\kappa^2 B^2 + \kappa^4) A^2}{B} + \frac{c_1 A^4}{3B} + \frac{8c_2 A^6}{45B}. \quad (18) \end{aligned}$$

2.3. Polynomial law

The CQ–CGLE with polynomial nonlinear form is structured as:

$$\begin{aligned} i q_t + i a q_{xxx} + b q_{xxxx} + (c_1 |q|^2 + c_2 |q|^4 + c_3 |q|^6) q \\ = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4 |q|^2 q^*} [2 |q|^2 (|q|^2)_{xx} - \{(|q|^2)_x\}^2] + \gamma q \\ + i \left[\begin{array}{c} \lambda(|q|^2 q)_x + \delta q_x + \nu |q|^2 q_x \\ + \mu(|q|^2)_x q \end{array} \right]. \quad (19) \end{aligned}$$

The structure of the bright soliton to the CQ–CGLE (19) would bear the same structure as in Kerr nonlinear form as given by (9).

$$\begin{aligned} T_3^t &= \frac{u b u_{xxxx}}{2} + \frac{v a u_{xxx}}{2} + \frac{v^2 c_1 u^2}{2} + \frac{v b v_{xxxx}}{2} \\ &- \frac{v \delta u_x}{2} + \frac{v^4 c_1}{4} + \frac{u^4 c_1}{4} - \frac{\nu v u_x u^2}{4} + \frac{v \mu u_x u^2}{4} \\ &- \frac{u \mu v_x v^2}{4} + \frac{u v v_x v^2}{4} + \frac{v^2 u^4 c_2}{2} + \frac{\nu v_x u^3}{4} + \frac{\mu u_x v^3}{4} \\ &- \frac{u a v_{xxx}}{2} + \frac{u \delta v_x}{2} - \frac{\nu u_x v^3}{4} - \frac{\mu v_x u^3}{4} + \frac{u^2 c_2 v^4}{2} \\ &+ \frac{v^2 u^6 c_3}{2} + \frac{3u^4 c_3 v^4}{4} + \frac{u^2 c_3 v^6}{2} - \frac{\gamma v^2}{2} \\ &- \frac{\gamma u^2}{2} + \frac{v^6 c_2}{6} + \frac{u^6 c_2}{6} + \frac{v^8 c_3}{8} + \frac{u^8 c_3}{8}, \quad (20) \end{aligned}$$

so that

$$\begin{aligned} \Phi_3^t &= \frac{b \mathcal{R}(q q_{xxxx}^*)}{2} - \frac{a \mathcal{J}(q^* q_{xxx})}{2} + \frac{c_1 |q|^4}{4} \\ &+ \frac{c_2 |q|^6}{6} + \frac{c_3 |q|^8}{8} - \frac{\gamma |q|^2}{2} - \frac{\mu \mathcal{J}(q^* q_x) |q|^2}{4} \\ &+ \frac{\delta \mathcal{J}(q^* q_x)}{2} + \frac{\nu \mathcal{J}(q^* q_x) |q|^2}{4}. \quad (21) \end{aligned}$$

The Hamiltonian is therefore given as:

$$\begin{aligned} H &= \int_{-\infty}^{\infty} \Phi_3^t dx \\ &= \int_{-\infty}^{\infty} \left[\frac{a(q_x^* q_{xx} - q_x q_{xx}^*)}{4i} + \frac{b |q_{xx}|^2}{2} + \frac{c_1 |q|^4}{4} \right. \\ &+ \frac{c_2 |q|^6}{6} + \frac{c_3 |q|^8}{8} - \frac{\gamma |q|^2}{2} + \frac{\delta(q^* q_x - q q_x^*)}{4i} \\ &\left. - \frac{(\mu - \nu)(q^* q_x - q q_x^*) |q|^2}{8i} \right] dx \end{aligned}$$

$$\begin{aligned}
& -\frac{(\mu - \nu)(q^* q_x - q q_x^*)|q|^2}{8i} dx \\
& = \frac{\kappa A^4(\mu - \nu)}{3B} - \frac{A^2(\delta\kappa + \gamma)}{B} \\
& - \frac{2a\kappa A^2(B^2 + \kappa^2)}{B} + \frac{bA^2(7B^4 + 2\kappa^2 B^2 + \kappa^4)}{B} \\
& + \frac{c_1 A^4}{3B} + \frac{8c_2 A^6}{45B} + \frac{4c_3 A^8}{35B}. \quad (22)
\end{aligned}$$

2.4. Quadratic–cubic law

The CQ–CGLE with quadratic–cubic nonlinear form is formulated to be:

$$\begin{aligned}
i q_t + i a q_{xxx} + b q_{xxxx} + (c_1 |q| + c_2 |q|^2) q &= \alpha \frac{|q_x|^2}{q^*} \\
+ \frac{\beta}{4|q|^2 q^*} [2|q|^2 (|q|^2)_{xx} - \{(|q|^2)_x \}^2] + \gamma q \\
+ i \left[\frac{\lambda (|q|^2 q)_x + \delta q_x + \nu |q|^2 q_x}{+\mu (|q|^2)_x q} \right]. \quad (23)
\end{aligned}$$

The bright soliton to the CQ–CGLE (23) is structured as [2]:

$$q(x, t) = \text{Asech}^2[B(x - vt)] e^{i(-\kappa x + \omega t + \theta_0)}, \quad (24)$$

$$\begin{aligned}
T_3^t &= -\frac{1}{12\sqrt{v^2 + u^2}} \left[-3\sqrt{v^2 + u^2} c_2 v^4 \right. \\
& - 3\sqrt{v^2 + u^2} \mu u_x v^3 + 3\sqrt{v^2 + u^2} \nu u_x v^3 \\
& + 3u\sqrt{v^2 + u^2} \mu v_x v^2 - 3\sqrt{v^2 + u^2} \nu v_x u^3 \\
& - 3u\sqrt{v^2 + u^2} \nu v_x v^2 + 3\sqrt{v^2 + u^2} \mu v_x u^3 \\
& - 6\sqrt{v^2 + u^2} v^2 c_2 u^2 - 3v\sqrt{v^2 + u^2} \mu u_x u^2 \\
& - 3\sqrt{v^2 + u^2} c_2 u^4 + 3v\sqrt{v^2 + u^2} \nu u_x u^2 - 4v^4 c_1 \\
& - 8v^2 c_1 u^2 - 4u^4 c_1 - 6v\sqrt{v^2 + u^2} a u_{xxx} \\
& - 6v\sqrt{v^2 + u^2} b v_{xxxx} - 6u\sqrt{v^2 + u^2} b u_{xxxx} \\
& + 6\sqrt{v^2 + u^2} \gamma v^2 + 6v\sqrt{v^2 + u^2} \delta u_x \\
& + 6\sqrt{v^2 + u^2} \gamma u^2 \\
& \left. - 6u\sqrt{v^2 + u^2} \delta v_x + 6u\sqrt{v^2 + u^2} a v_{xxx} \right], \quad (25)
\end{aligned}$$

so that

$$\begin{aligned}
\Phi_3^t &= -\frac{1}{12|q|} [-6b\mathcal{R}(qq_{xxxx})|q| + 6a\mathcal{I}(q^* q_{xxx})|q| \\
& - 4c_1|q|^4 - 3c_2|q|^5 + 6\gamma|q|^3 + 3\mu\mathcal{I}(q^* q_x)|q|^3 \\
& - 6\delta\mathcal{I}(q^* q_x)|q| - 3\nu\mathcal{I}(q^* q_x)|q|^3]. \quad (26)
\end{aligned}$$

The three conserved quantities are:

$$P = \int_{-\infty}^{\infty} |q|^2 dx = \frac{4A^2}{3B}, \quad (27)$$

$$M = ia \int_{-\infty}^{\infty} (q^* q_x - q q_x^*) dx = \frac{4\kappa A^2}{3B}, \quad (28)$$

$$H = \int_{-\infty}^{\infty} \Phi_3^t dx$$

$$\begin{aligned}
& = \int_{-\infty}^{\infty} \left[\frac{a(q_x^* q_{xx} - q_x q_{xx}^*)}{4i} + \frac{b|q_{xxx}|^2}{2} + \frac{c_1|q|^3}{3} \right. \\
& + \frac{c_2|q|^4}{4} - \frac{\gamma|q|^2}{2} + \frac{\delta(q^* q_x - q q_x^*)}{4i} \\
& \left. - \frac{(\mu - \nu)(q^* q_x - q q_x^*)|q|^2}{8i} \right] dx \\
& = \frac{8(\mu - \nu)\kappa A^4}{35B} - \frac{2A^2(\delta\kappa - \gamma)}{3B} \\
& + \frac{16c_1 A^3}{45B} + \frac{2b(80B^4 + 168\kappa^2 B^2 + 35\kappa^4)A^2}{105B} \\
& - \frac{4a\kappa(B^2 + 5\kappa^2)A^2}{15B} + \frac{8c_2 A^4}{35B}. \quad (29)
\end{aligned}$$

2.5. Parabolic–nonlocal law

The CQ–CGLE with parabolic–nonlocal nonlinear form is given as:

$$\begin{aligned}
i q_t + i a q_{xxx} + b q_{xxxx} \\
+ [c_1 |q|^2 + c_2 |q|^4 + c_3 (|q|^2)_{xx}] q \\
= \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*} [2|q|^2 (|q|^2)_{xx} - \{(|q|^2)_x \}^2] + \gamma q \\
+ i \left[\frac{\lambda (|q|^2 q)_x + \delta q_x + \nu |q|^2 q_x}{+\mu (|q|^2)_x q} \right]. \quad (30)
\end{aligned}$$

The bright soliton is structured the same as that of Kerr nonlinear form and hence the power stays the same as that of Kerr nonlinear form as given by (12). However, in this case, there are no momentum and Hamiltonian densities. Therefore, linear momentum and Hamiltonian does not exist in this case.

3. Conclusions

This work is about conservation laws that emerged from CQ–CGLE that was considered with five nonlinear forms. The Hamiltonian, linear momentum and the power conserved quantities are enlisted for such nonlinear forms, save the parabolic–nonlocal form that comes with one conservation law, namely the power. These conservation laws imply that the soliton perturbation theory can be immediately pursued and thus the quasi–monochromatic dynamics of such solitons are retrievable. Next, the quasi–stationary solitons for CQ–CGLE are to be found. These are all on the horizon

References

- [1] G. Akram, N. Mahak, *Optik* **164**, 210 (2018).
- [2] A. Biswas, Y. Yildirim, M. Ekici, P. Guggilla, S. Khan, O. Gonzalez–Gaxiola, A. Khamis Alzahrani, M. R. Belic, *J. Appl. Sci. Eng.* **24**(6), 937 (2021).
- [3] A. Biswas, A. H. Kara, Y. Sun, Q. Zhou, Y. Yildirim, H. M. Alshehri, M. R. Belic, *Results Phys.* **31**, 104901 (2021).
- [4] C. Huang, Z. Li, *Math. Probl. Eng.* **2021**, 6640086 (2021).
- [5] N. A. Kudryashov, *Appl. Math. Comput.* **386**, 125407 (2020).
- [6] D. S. Mou, J. J. Fang, Y. Fan, *Optik* **244**, 167554 (2021).
- [7] S. Naghshband, M. A. F. Araghi, *Ain Shams Eng. J.* **9**(4), 607 (2018).
- [8] Y. Qiu, B. A. Malomed, D. Mihalache, X. Zhu, J. Peng, Y. He, *Phys. Lett. A* **383**(228), 2579 (2019).
- [9] H. Sakaguchi, B. Malomed, *J. Phys. Soc. Japan* **72**, 1360 (2003).
- [10] S. Shwetanshumala, *Prog. Electromagn. Res. Lett.* **3**, 17 (2008).
- [11] A. M. Wazwaz, *Appl. Math. Lett.* **19**(10), 1007 (2006).
- [12] Y. Yan, W. Liu, *Appl. Math. Lett.* **98**, 171 (2019).
- [13] Z. Yan, K. W. Chow, B. A. Malomed, *Chaos Solit. Fractals* **42**(5), 3013 (2009).
- [14] E. M. E. Zayed, M. E. M. Alngar, A. Biswas, M. Ekici, S. Khan, A. S. Alshomrani, *J. Commun. Technol. Electron.* **66**(5), 481 (2021).
- [15] Y. Zhao, C. Y. Xia, H. B. Zeng, *Phys. Lett. A* **384**(18), 126395 (2020).

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